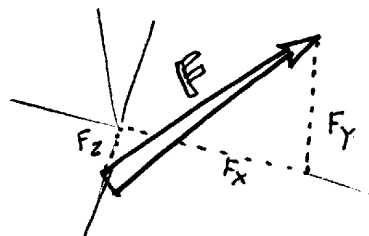


STATICS REVIEW



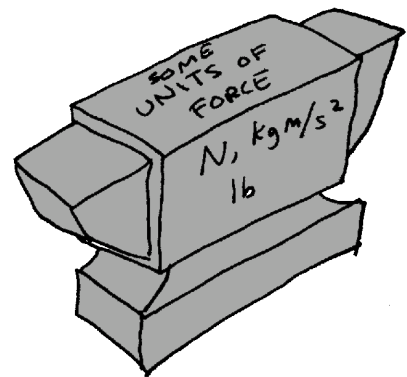
STATICS APPLIES WHEN STRUCTURES/RIGID BODIES ARE IN EQUILIBRIUM.

A **FORCE** IS A VECTOR QUANTITY, HAS A MAGNITUDE, POINT OF APPLICATION, AND A DIRECTION.

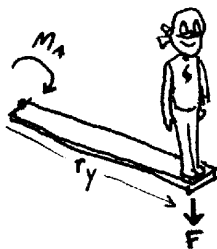


$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{k}$$

In statics, $\sum F_x = 0$
 $\sum F_y = 0$
 $\sum F_z = 0$



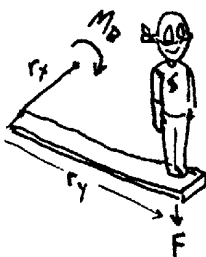
A **MOMENT** IS A CROSS PRODUCT OF A RADIUS VECTOR $\mathbf{r}$ AND A FORCE $\mathbf{F}$



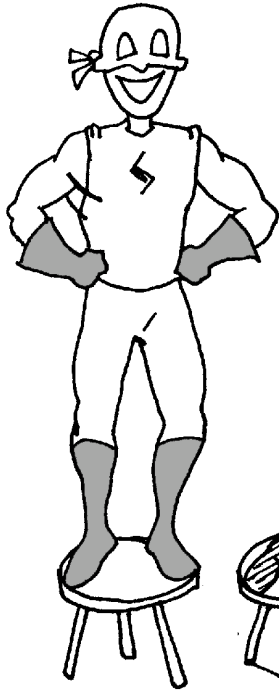
$$\mathbf{M} = \mathbf{r} \times \mathbf{F} \quad \mathbf{M} = M_x \mathbf{i} + M_y \mathbf{j} + M_z \mathbf{k}$$

In statics, $\sum M_x = 0$
 $\sum M_y = 0$
 $\sum M_z = 0$

$M_A = M_B$, these moments are the same!



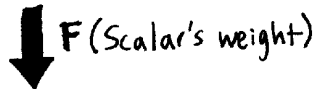
STRESS & STRAIN



STRESS IS A FORCE/AREA

$$\sigma = F/A$$

units are N/m^2 , Psi, Pa



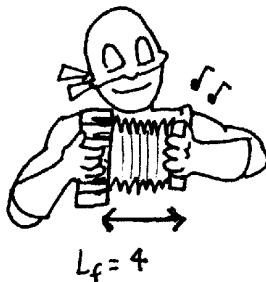
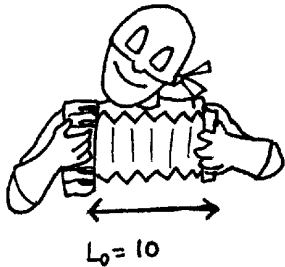
The stress here is less than the stress at A_2 because of the difference in Area.



I'd like to get this stress off my back!

STRAIN HAS NO UNITS. IT'S THE DIFFERENCE IN LENGTH OVER THE ORIGINAL LENGTH

$$\epsilon = \frac{L_f - L_o}{L_o}$$



$$\epsilon = \frac{4 - 10}{10} = -\frac{3}{5}$$



Don't strain yourself Vector!

©AlisonWong2005

$$E = \frac{\sigma}{\epsilon}$$

NOW, WHEN YOU PUT σ/ϵ , YOU GET THE

ELASTIC MODULUS (A.K.A. YOUNG'S MODULUS)

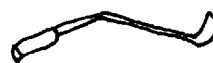
This E differs for every material and it gives you an idea of its relative stiffness.



diamond (high E)
1000 GPa

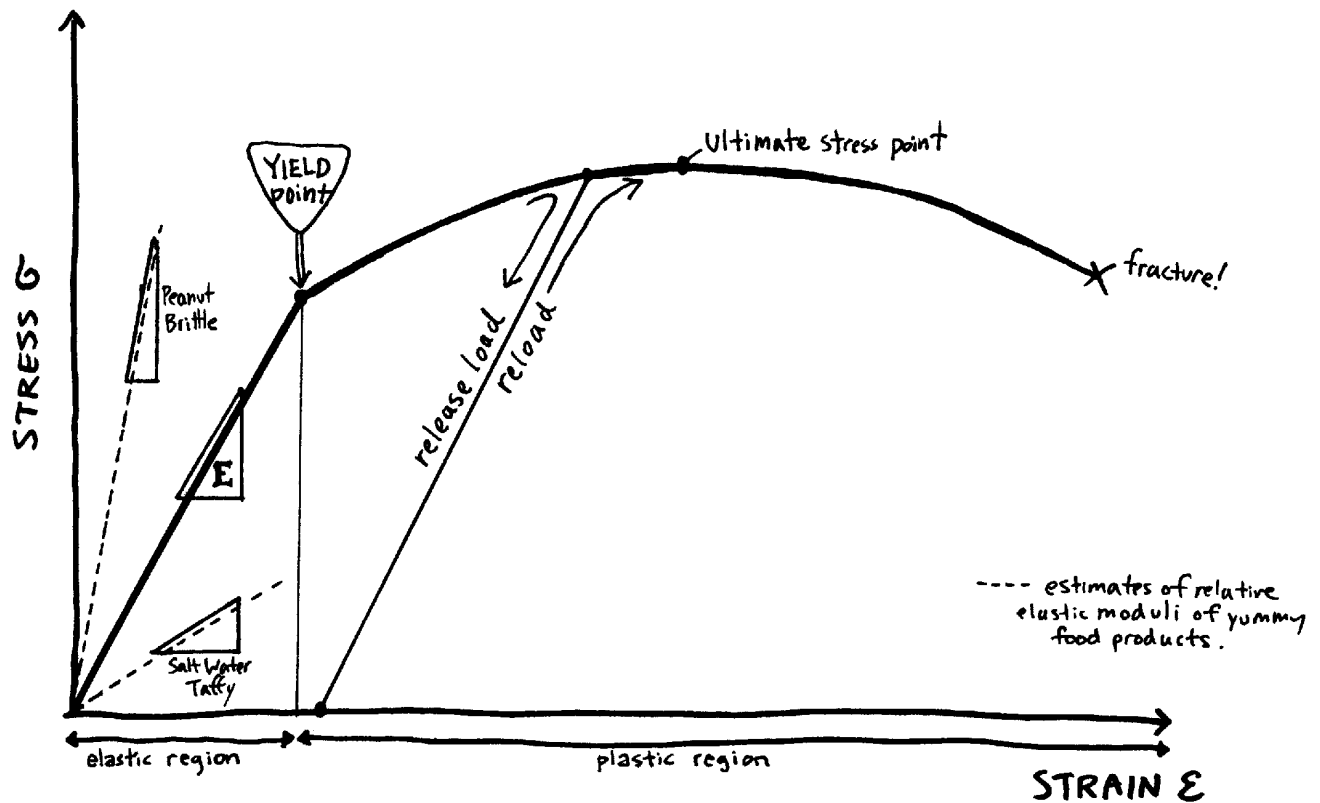


taffy (low E)
.01 GPa



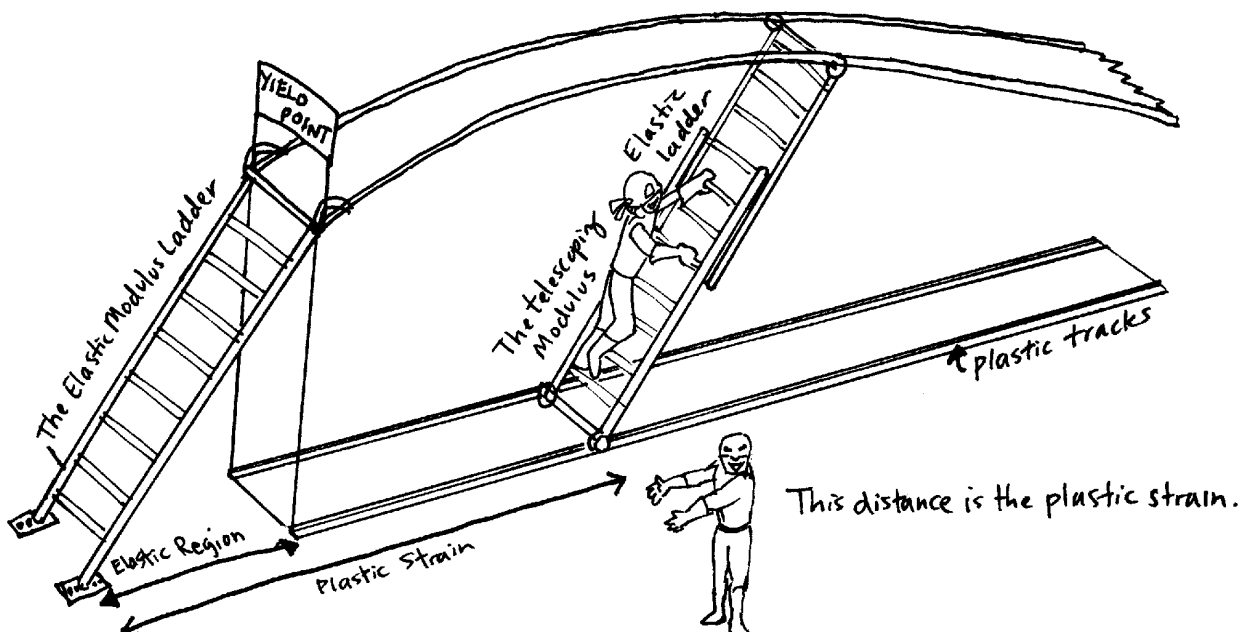
Aluminum putter (low E)
69 GPa

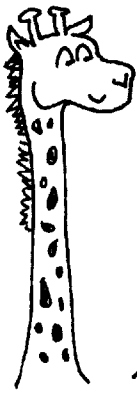
STRESS STRAIN DIAGRAM



The slope $\frac{\sigma}{\epsilon}$ is the elastic modulus! (Thus, taffy would have a smaller slope than steel.)

During the elastic region, the material bounces back to its original shape when a load is released. However, after a certain stress is reached (the yield point) the material falls back with the same slope, but ends at a strain that's NOT ZERO! That's permanent deformation (A.K.A plastic strain)





NECKING HAPPENS WHEN THE CROSS-SECTIONAL AREA DECREASES AFTER THE ULTIMATE STRESS



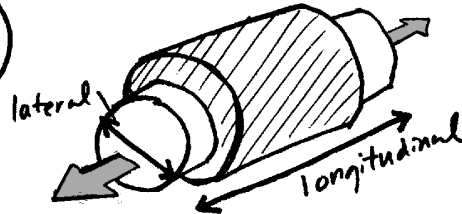
DUCTILE materials can undergo large plastic deformations before fracture

BRITTLE materials fracture with little or no deformation

STRAIN (or WORK) HARDENING is when the yield point is at a higher stress level than the original yield point. Basically, from the previous page, Scalar's ladder doesn't change in slope, but it changes in height after the original yield point. Whenever the ladder is higher than the original ladder is where the yield point is higher (that's where strain hardening occurs)

MATERIAL PROPERTIES

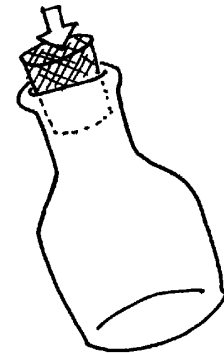
POISSON'S RATIO $\nu = - \left(\frac{\epsilon_{\text{lateral}}}{\epsilon_{\text{longitudinal}}} \right)$



Longitudinal strain is parallel to the load whereas lateral strain is perpendicular to the load.

ν gives you an idea of how compressible a material is.

Something that has $\nu = .5$ is incompressible. That means that the VOLUME stays constant even if it is pushed and pulled in many directions. Cork is close to 0, so it doesn't swell outwards when force is applied on top. (axially)



FACTOR OF SAFETY

$$K_{\text{safety}} = \left(\frac{\text{Value that produces failure}}{\text{Computed Value}} \right)$$

$$= \left(\frac{\text{maximum}}{\text{what's allowed}} \right)$$



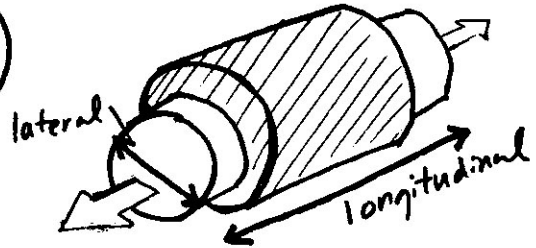
Scalar, if 10 donuts make you sick, and your K_s is 2, you're only allowed to eat 5!

MATERIAL PROPERTIES

POISSON'S RATIO

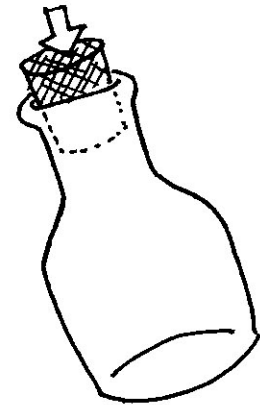
$$\nu = - \left(\frac{\epsilon_{\text{lateral}}}{\epsilon_{\text{longitudinal}}} \right)$$

Pronounced
= "N-U"



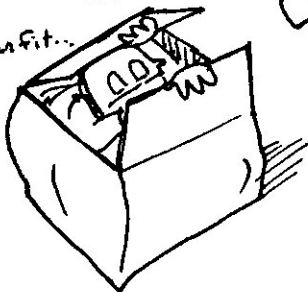
Longitudinal strain is parallel to the load whereas lateral strain is perpendicular to the load.

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That's NOT
what I meant by
compressible!

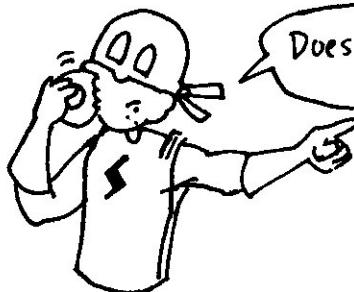
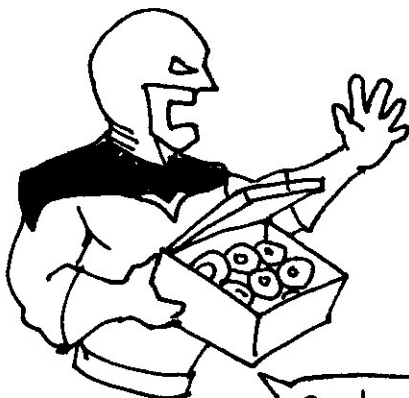
I can fit...



FACTOR OF SAFETY

$$K_{\text{Safety}} = \left(\frac{\text{Value that produces failure}}{\text{Computed Value}} \right)$$

$$= \left(\frac{\text{maximum}}{\text{what's allowed}} \right)$$



Does that include the pie
I ate earlier?

Scalar, if 10 donuts make you sick, and your K_s is 2, you're only allowed to eat 5!

EFFECT OF TEMPERATURE

$$\epsilon_T = \frac{\Delta L}{L} = \alpha \Delta T$$

This is the strain that results from a change in temperature.
 α is the temperature coefficient of the material.

AXIAL FORMULA

$$\Delta L = \frac{PL}{AE}$$

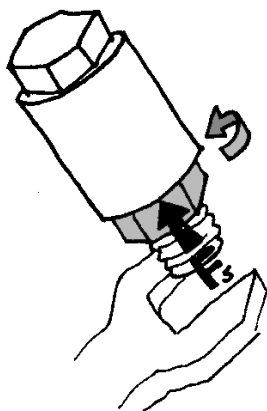
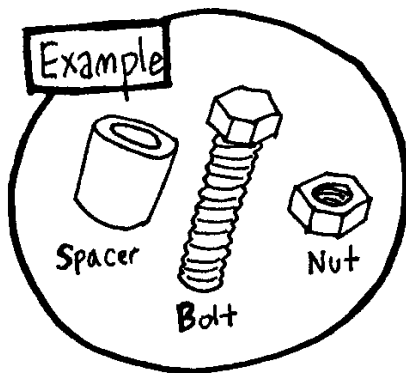
Use this to see how much a material shortens or lengthens from an applied load, P . A is the cross sectional area, E is the Young's modulus, and L is the original length.



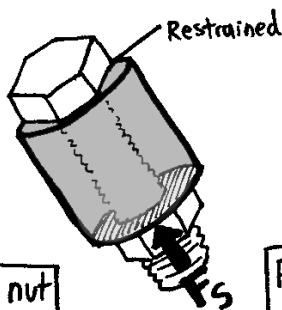
Vector, what's my α ? I wonder how much my head will shrink!



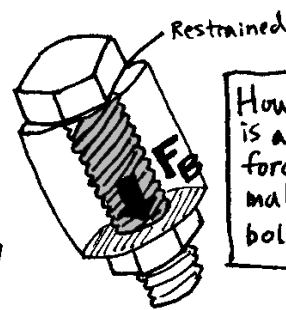
Maybe I should go take you to see one instead!



Turning nut results in force F_s



F_s compresses spacer ΔL_s



However, there is a reactive force, F_B , that makes the bolt lengthen!

$$F_s = F_B$$

$$\Delta L = \frac{PL}{EA}$$

$$\Delta L_{SPACER} = \frac{F_s L_s}{E_s A_s}$$

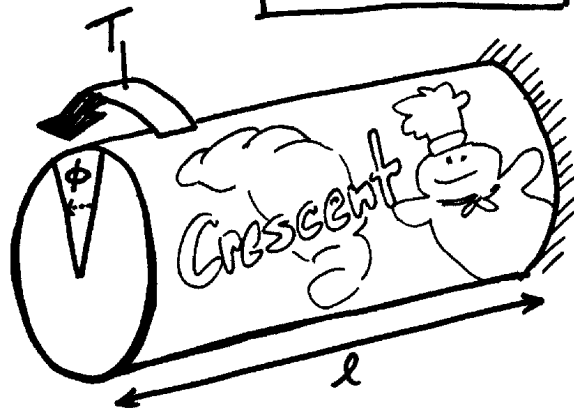
$$\Delta L_{BOLT} = \frac{F_B L_B}{E_B A_B}$$

TORSION

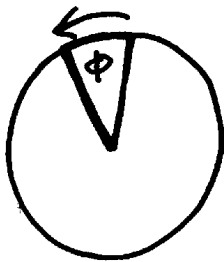


$$\phi = \frac{Tl}{GJ}$$

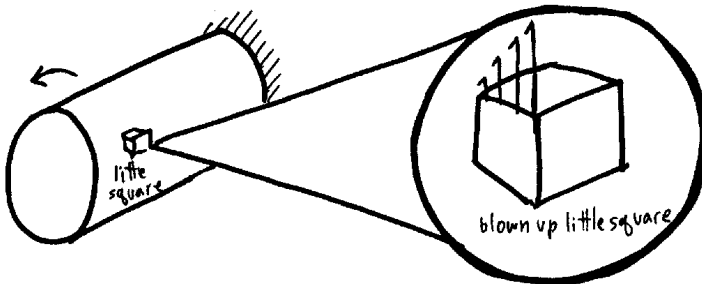
where T is the torque applied, l is the length of the circular shaft, G is the shear modulus, and J is the polar moment of inertia.



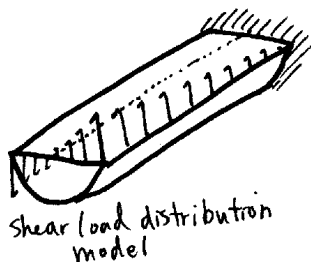
When one side of a circular shaft is constrained, the Torque applied to the other end causes a change of angle ϕ .



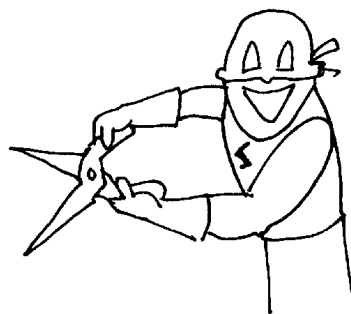
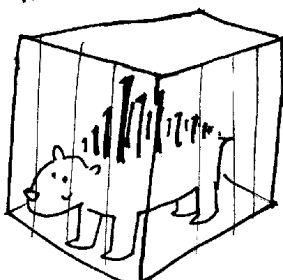
An element on the outside radius will move more than an element closer in. This corresponds to the inner shear stress. Think of a record - the outside edge spins at a greater velocity than the center, but the angular velocity is the same.



Notice that the shear stress is greater at the edge. Also, there is NO AXIAL STRESS



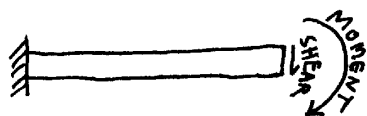
Notice that shear stress decreases as you go inward toward the center as well as when you go back towards the constrained end.



Meet Shear Stress porcupine!



SHEAR & MOMENT DIAGRAMS

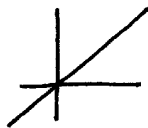


WHY?

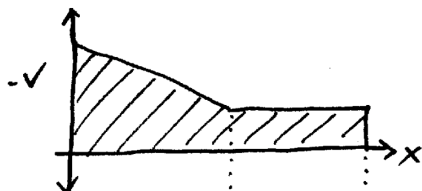
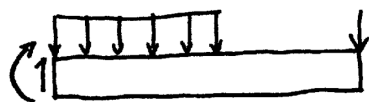
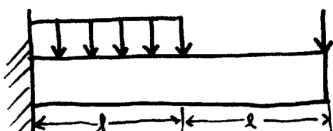
BY PLOTTING THESE DIAGRAM, WE CAN SEE WHERE THE MAX VALUES OF SHEAR FORCE AND BENDING MOMENT OCCUR

$V(x)$ = SHEAR FORCE CURVE

i.e.



$M(x)$ = BENDING MOMENT CURVE
(integral of $V(x)$)



1 THINK OF THE FREE BODY DIAGRAM
Where are the forces, reaction forces, and moments?



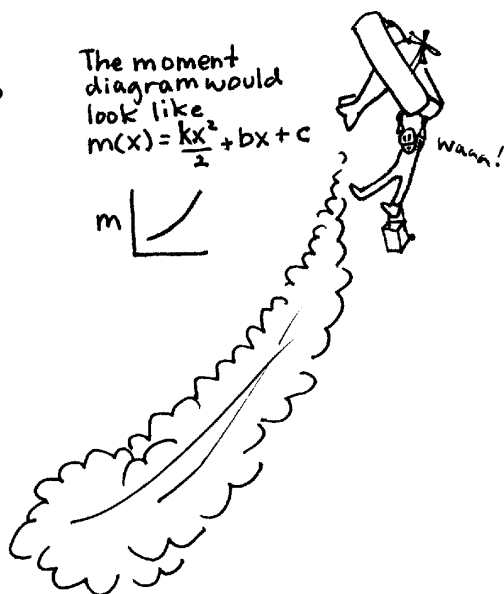
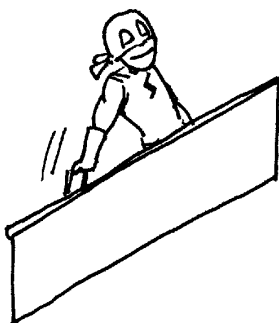
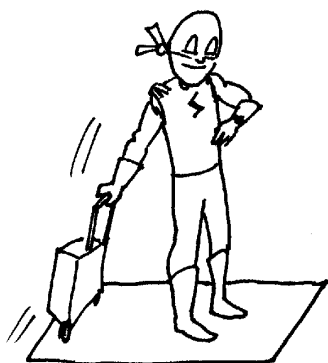
2 PLOT AND SKETCH DISTRIBUTED LOADS
The shear force curve is ONE ORDER HIGHER than the load curve. The moment curve is ONE ORDER HIGHER than the shear force curve.

if you have a distributed load $\uparrow\uparrow\uparrow F(x)=k$

The Shear diagram would look like $V(x)=kx+b$

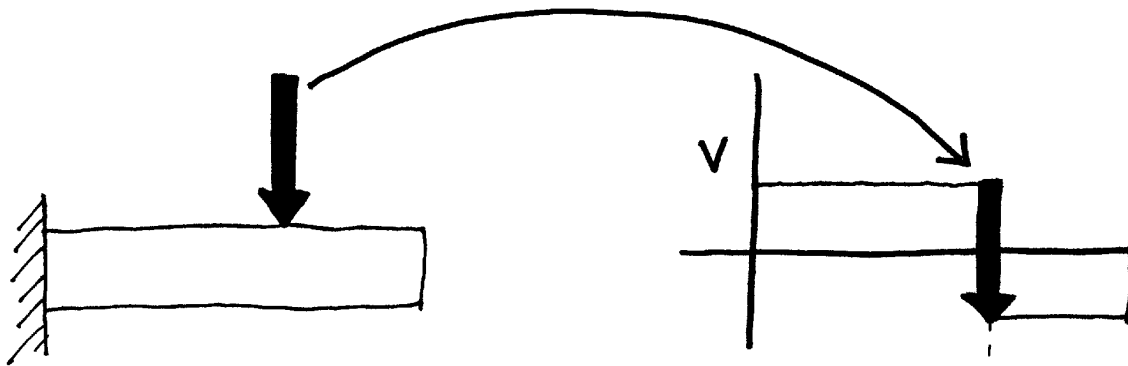


The moment diagram would look like $m(x)=\frac{kx^2}{2}+bx+c$

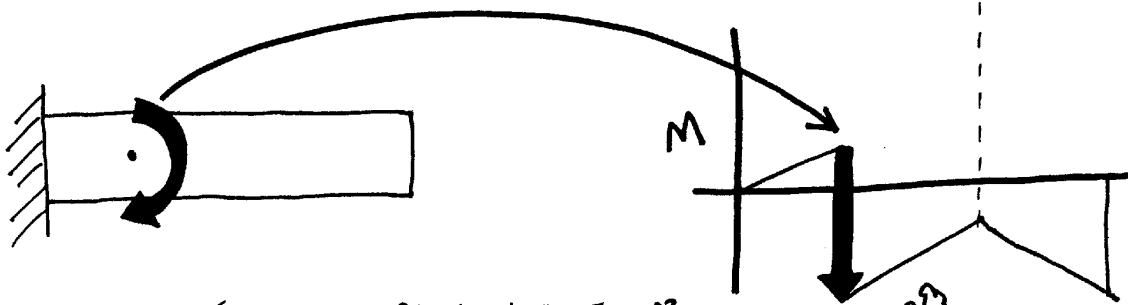


3

PLOT AND SKETCH POINT LOADS AND MOMENTS.

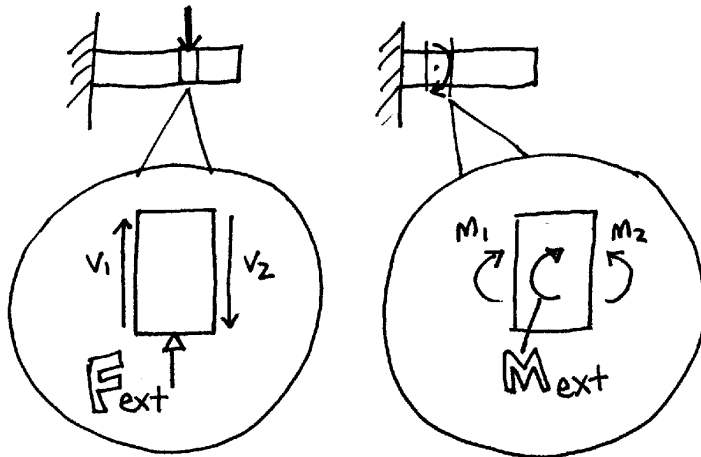


A POINT LOAD CAUSES THE SHEAR FORCE TO JUMP BY THAT VALUE IN THE DIAGRAM. A POINT MOMENT WILL DO THE SAME IN MOMENT DIAGRAM.



HOW TO CHOOSE WHICH DIRECTION TO JUMP?

1. ISOLATE THE AREA WHERE THE POINT LOAD/MOMENT IS.



2. $V_2 = V_1 + F_{ext}$

$M_2 = M_1 + M_{ext}$

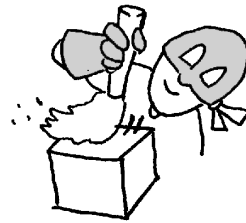
USE THE EQUATIONS TO DETERMINE F_{ext} OR M_{ext}

THE JUMP IN THE DIAGRAM IS IN THE DIRECTION OF F_{ext} OR M_{ext}



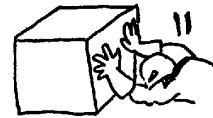
THE JOY OF STRESS TRANSFORMATION

BROUGHT TO YOU BY VECTOR & SCALAR



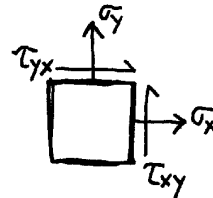
SHEAR STRESS

τ

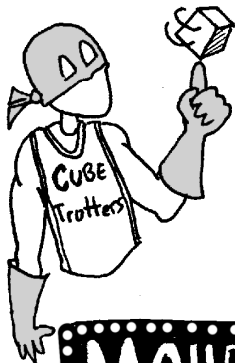


AXIAL STRESS

σ



REMEMBER,
 $\tau_{xy} = \tau_{yx}$



THIS CUBE CAN BE
ROTATED IN MANY
DIRECTIONS - WE WANT
TO FIND THE DIRECTION
AND VALUES WHEN
THE STRESSES ARE

MAX

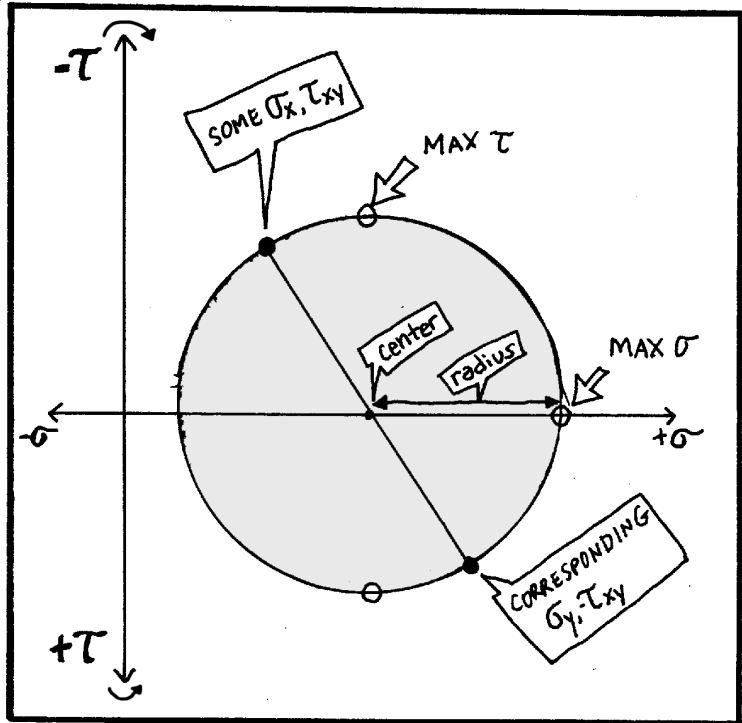
(a.k.a. principal stresses!)

MOHR'S CIRCLE

IT HAS A τ AND σ AXIS.

(σ_x, τ_{xy}) AND (σ_y, τ_{yx}) ARE POINTS
ON THE CIRCLE.

YOU JUST NEED THE **RADIUS** AND
THE **CENTER** AND YOU'RE ALL SET!



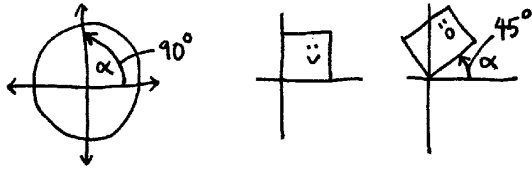
$$\text{RADIUS} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\text{MAX } \sigma = (\text{CENTER} + \text{RADIUS}) \text{ OR } (\text{CENTER} - \text{RADIUS})$$

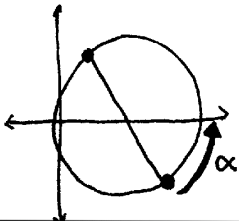
$$\text{MAX } \tau = (\text{RADIUS})$$

$$\text{CENTER} = \left(\frac{\sigma_x + \sigma_y}{2}\right)$$

REMEMBER, MOVING θ IN THE PHYSICAL WORLD
TRANSLATES INTO 2θ IN THE MOHR CIRCLE WORLD.



LET'S SAY YOUR GIVEN STRESSES LEAD YOU TO
THIS PLOT... AND YOU WANT TO KNOW WHAT
ORIENTATION GIVES YOU MAX σ
STRESS.... WHY, IT'S JUST
WHAT THE GRAPH SHOWS, α !
(BUT IT WOULD BE $\alpha/2$ IN THE
REAL WORLD!)



YOU MUST CHOOSE
BETWEEN THE θ
RED PILL AND
THE 2θ BLUE PILL



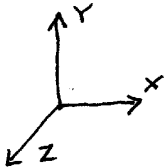
AND DON'T
FORGET
THE

**3D MOHR
CIRCLE**



YOU'VE GOT 3 PLANES RIGHT?

SO... YOU NEED 3 CIRCLES!



YOU BASICALLY ONLY NEED TO
KNOW 2 CIRCLES' INFO, THEN
YOU CAN FIND THE THIRD'S.
THE 3D MOHR CIRCLE USES
THE SAME AXIS AS 2D.
IT JUST HAS "MOHR"
CIRCLES. (hm hm)

